A delayed payment method in co-ordinating a single-vendor multi-buyer supply chain

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A B S T R A C T
In this paper, we consider a co-ordinated single-vendor multi-buyer supply chain model by synchronizing ordering and production cycles with delayed payments that are based on the buyers’ order intervals. It can be shown that the synchronized cycles policy works better, in terms of total system costs, than independent optimization. While the vendor is benefited from the co-ordination by synchronized cycles, this research proposes a co-ordination mechanism that incorporates a delayed payment method which can guarantee that a buyer’s total relevant cost of co-ordination will not be increased when compared with independent optimization. Most importantly, the vendor does not require any cost information from the buyers when applying this co-ordination mechanism.

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1. Introduction
Co-ordination between various parties in a supply chain is essential in nowadays business environments. An effective supply chain co-ordination can reduce average holding inventory level and the total expected cost. Various integrated inventory co-ordination models are established dated back in the 1970s (Sarmah et al., 2006; Khouja and Goyal, 2008). Under the traditional economic order quantity (EOQ) model assumptions, it is assumed that the buyers are required to pay the full amount of the items at the moment the items are delivered to the buyers. However, this may not be true in reality. In practice, the supplier will offer the buyers a delayed payment period, or the trade credit period, in paying for the amount of purchasing cost. The delayed payment method may be regarded as an alternative to price discounts to induce larger orders.

Pioneer researchers such as Haley and Higgins (1973), Chapman et al. (1984), and Goyal (1985) incorporated the delayed payment method into the classical inventory supply chain model. Rachamadugu (1989) analyzed the relationship between the length of the trade credit period and the EOQ via discounted cashflow approach. Kim et al. (1995) formulated a single-vendor single-buyer supply chain model with price elasticity demand function. The objective is to determine the optimal trade credit period that maximize the supplier’s own profit. Shinn (1997) simultaneously determined the retail price and the replenishment cycle time of the buyer with predetermined credit period set by the supplier with constant price elasticity demand function. Hwang and Shinn (1997) considered the lot-sizing problem with price-sensitive demand with exponentially deteriorating item. Jamal et al. (1997) considered the model with a deteriorating item and allowable shortages with fixed delayed payment period. Extending the model in Jamal et al. (1997), Jamal et al. (2000) considered the optimal payment time within a maximum permissible credit period to settle the payment of the orders. While Goyal (1985), implicitly, assumed infinite replenishment rate, Chung and Huang (2003) extended the model with finite replenishment rate. Abad and Jaggi (2003) formulated a co-ordinated model to determine the sales price and the length of the credit period simultaneously. Chung and Liao (2004) and Chung et al. (2005) considered models such that the offer of the trade credit period depends on the quantity ordered. If the ordering quantity is less than the level of predetermined quantity, no credit period will be offered. Otherwise, a fixed permissible delay of payment period is offered. Yang and Wei (2006) determined the replenishment policy with trade credit policy, finite replenishment rate and price sensitive demand for a deteriorating item. Jaber and Osmar (2006) also considered the integrated model under trade credit policy with profit sharing scheme. Jaggi et al. (2008) formulated a model such that the demand function depends on the length of the credit period, which is one of the variables to be optimized. Teng et al. (2007) considered a trade credit model in which the supplier offers a permissible delay period to the retailer, and the retailer offers a permissible delay period to its customers. Goyal and Chang (2008) considered a
model with the feasibility of permitting a one-time extension (fixed) credit period in settling the payments beyond the normal (fixed) credit period.

While most of the above researches considered single-vendor single-buyer supply chain system, recently Sarmah et al. (2008) incorporated the credit option concept to the common cycle model developed by Banerjee and Burton (1994) in a single-vendor multi-buyer supply chain. Under the common cycle model, all the buyers are replenished at the same predetermined cycle. Since the deliveries to the buyers can be made at the same time, the order processing and shipment costs can be reduced and thus result at a lower system cost.

As pointed out by Sarmah et al. (2006), one of the major limitations in the literature of co-ordinated supply chain models is that most of the models assume that a supply chain partner has complete information (including cost) about the other partner. In a recent review on trade credit policy (Chang et al. 2008), the authors also commented that most of the models in the literature require buyer’s cost information in determining the optimal values of the decision variables of interest, i.e. the replenishment cycles, the delayed payment periods, etc. Even the buyers are willing to reveal their cost information, the credibility of the shared information is still a major concern.

The motivation of this paper is to address the above major limitation of supply chain co-ordination models in the literature. Chan and Kingsman (2007) developed a synchronized cycles model which out-performs the common cycle policy as well as independent optimization in a number of numerical examples with different ranges of demands/cost parameters and different number of buyers. This paper proposes to incorporate a delayed payment period and a cost-sharing scheme into the synchronized cycles model so as to guarantee that every buyer will not be worse off when compared with independent optimization. The delayed payment period and the cost-sharing scheme are incentives offered by the vendor to motivate the buyers to participate in the co-ordination. More importantly, the proposed incentives do not require any of the buyers’ cost information. In addition, the delayed payment period proposed in our model is different for each buyer such that the savings achieved from the co-ordination can be shared in an equitable sense. In Sarmah et al. (2008) and others, the vendor offers a common period that is acceptable to all the buyers. That is, some of the buyers may obtain a longer period than it is required. Hence, some buyers may obtain more benefits from the co-ordination than the others. In this paper, we adopt varying delayed payment periods so as to guarantee that each buyer will have the same cost as in independent optimization.

The remaining of the paper is organized as follows: Section 2 provides a brief background on the independent policy and the synchronized cycles model. Section 3 outlines the cost change of buyers when they participate in the co-ordination. Section 4 incorporates the delay payment method together with the cost sharing scheme. Section 5 proposes an equitable scheme on profit sharing. Section 6 presents the results and followed by the conclusion in Section 7.

2. The independent policy and the synchronized cycles model

2.1. The independent policy

We assume that each of the \( n \) buyers faces a deterministic demand at rate \( d_i \), per unit time, incurs an ordering cost \( A_i \), each time it places an order and incurs an inventory holding cost \( h_i \) per unit per unit time held. If the buyers and the supplier operate independently then each buyer will order at time intervals of \( T_i \) units apart. The total costs per unit time for the \( i \)th buyer, \( B_i \), can thus be expressed as

\[
B_i = \frac{A_i}{T_i} + \frac{h_i d_i T_i}{2}.
\]  (1)

The economic order interval and economic order quantity is given by

\[
T_i^* = \frac{2A_i}{h_i d_i},
\]

and

\[
Q_i^* = d_i T_i^* = \sqrt{\frac{2A_1}{h_i}},
\]

respectively and the minimum total cost \( B_i^{\text{IND}} \) for the \( i \)th buyer is given by

\[
B_i^{\text{IND}} = \sqrt{2A_1 h_i d_i}.
\]  (2)

The vendor produces items at a rate of \( P \) per unit time with \( P > D \). We assume that the vendor incurs a set-up cost \( S_i \) for each production run, incurs a holding cost of \( h_i \) per unit per unit time and \( C_i \) denotes the order processing and shipment cost per order. Clearly, in the situation where the vendor and the buyers are operating independently, the vendor needs to carry a large stock of items to satisfy all demands on time, or the buyers will have to suffer stock outs and/or late deliveries. The largest possible aggregate ordering size is \( \sum_{i=1}^{n} Q_i \) units of stock when all the buyers replenish at the same time. The total cost function, per unit time, incurred by the vendor is expressed as

\[
V^{\text{IND}}(Q_i) = \frac{S_i D}{Q_i} + h Q_i \left( 1 - \frac{D}{P} \right) + \sum_{i=1}^{n} \frac{C_i d_i}{Q_i} + h_i \sum_{i=1}^{n} Q_i^*.
\]  (4)

By standard inventory model, the optimal economic batch production \( Q_i^* \) is

\[
Q_i^* = \sqrt{2S_i D \frac{h_i}{P(1 - \frac{D}{P})}}.
\]

From (2) and (4), it can be shown that the total system cost for the independent policy is

\[
T^{\text{IND}} = \sqrt{2S_i h D \left( 1 - \frac{D}{P} \right) + \sum_{i=1}^{n} \left[ \frac{2A_i h_i + 2A_i h_i + C_i h_i}{\sqrt{2A_i h_i}} \right]}.
\]  (6)

2.2. The synchronized cycles model

Co-ordinating the timing of deliveries of the buyers with the production policy of the vendor may enable the vendor to avoid stockouts. Banerjee and Burton (1994) and others proposed that the buyers should adopt a common order cycle of \( T \) periods apart. In order to meet these scheduled demands the vendor will have to adopt a production cycle that is some integer multiple of \( T \), say \( N \), where \( N \geq 1 \).

However, forcing all buyers to use the same common cycle time of \( T \) may be costly. It may be more economical to have shorter cycle times for the buyers having low demand and large cycle times for the buyers having high demand. The synchronized cycles model developed by Chan and Kingsman (2007) resolves the shortcoming in the common cycle model. In the synchronized cycles model, given a basic cycle time \( T \), the vendor chooses his production cycle as an integer multiple, \( N \), of the basic cycle time. For the buyers,
each of them is allowed to choose its ordering interval as an integer factor, $k_i$, of the vendor's production cycle time.

For simplicity we assume that delivery to the buyers is instantaneous, or more exactly that buyers' orders are received and deducted from the vendor's inventory at regular intervals $T$ apart. The result of the co-ordination will be a set of demands $D_1, D_2, \ldots, D_n$ over the $NT$ periods of the vendor production cycle, where each demand is some subset of the buyers' order quantities. To determine the vendor's stockholding cost we first need to consider how to meet these given demands and then secondly consider how to allocate the individual buyer's orders to the successive demands $D_1, D_2, \ldots, D_n$.

Under the synchronized cycles model, which the buyer orders at an interval of $k_iT$, the cost function borne by the buyer is similar to (1) except that the variable $T_i$ is replaced by $k_iT$, i.e.

$$
B_i^{SAC} = A_i + \frac{h_i d_i k_i T}{2}.
$$

Again, let $S_r$ denotes the vendor's set-up cost per production run, $h$ denotes the holding cost per item per unit of time, and $C_i$ denotes the order processing and shipment cost per order. Since each buyer places orders at an interval of $k_iT$ and the vendor's production cycle is $NT$, the vendor's set-up cost and vendor's order processing and shipment cost per unit time is expressed as

$$
S_v \left\{ \frac{NT}{k_iT} \right\}
$$

and

$$
\sum_{i=1}^{n} C_i \left\{ \frac{k_i T}{NT} \right\}
$$

respectively.

If every buyer places orders as early as possible in each of the order cycles, the relevant vendor's holding cost can then be expressed as

$$
\left\{ \frac{h D - h D^2}{2} \right\} NT + \sum_{i=1}^{n} d_i \left\{ \frac{h D}{2} - \frac{1}{2} h \right\} k_i T.
$$


Summing up all the costs in (7)–(10) for all the buyers and the vendor, the total system cost is given by

$$
T^{SCA}_v = S_v \left\{ \frac{NT}{k_iT} \right\} + \sum_{i=1}^{n} \left\{ \frac{C_i + A_i}{k_iT} \right\} + \sum_{i=1}^{n} d_i \left\{ \frac{h D}{2} - \frac{1}{2} h \right\} k_i T.
$$

3. The increased cost of buyers

It is shown in Chan and Kingsman (2007) that the synchronized cycles model can be used to plan the ordering intervals in a one-vendor many-buyer supply chain so as to reduce significantly the system costs compared to each partner operating independently. The cost saving of the system is achieved by a significant reduction in the vendor's cost. However, the cost to all the buyers is significantly increased. This appears to be a general result that applies in all analyses of co-ordinated ordering, inventory, and production planning models. The vendor is motivated to seek to co-ordinate decisions in the whole supply chain but the buyers are not. Hence, the interest is in examining what mechanism is needed from the vendor to motivate the buyers to change their policies to allow the savings from co-ordination to be achieved.

Recall, from Eq. (2), that the cost of buyer $i$ in independent optimization is

$$
B_i^{IND} = \sqrt{2A_i h_i d_i}.
$$

From Eq. (7), the total cost of buyer $i$ in the synchronized cycles policy is

$$
B_i^{SCA} = A_i + \frac{h_i d_i k_i T}{2}.
$$

Hence, buyer $i$ has an increased cost of

$$
\frac{A_i}{k_i T} + \frac{h_i d_i k_i T}{2} = \left( \frac{T_i}{2} \right)^2 h_i d_i (\text{Note that } A_i = \frac{T_i}{2} h_i d_i)\frac{T_i}{2} k_i T (k_i T - 1) h_i d_i = \frac{1}{2} \left( 1 - \frac{T_i}{k_i T} \right)^2 h_i d_i k_i T.
$$

As Eq. (12) is always non-negative, this implies that the total relevant cost of the $i$th buyer under the co-ordinated system is always higher than that of independent optimization.

4. A co-ordination incentive: the delayed payment method with storage cost sharing

In Section 3, the proposed co-ordination mechanism requires the buyers to reveal the holding costs to the vendor. However, in practice, the buyer may not be willing to do so. Hence, in this section, we propose a co-ordination incentive that the vendor can provide for the buyers without acquiring the buyer's cost information.

Consider that the holding cost consists of two components, the capital opportunity cost ($h_i^c$ and $h_i^f$) and the storage cost ($h_i^s$ and $h_i^m$). Then, the buyer's and the vendor's holding cost can be, respectively, expressed as

$$
h_i = h_i^c + h_i^m \text{ and } h = h_i^c + h_i^m.
$$
In this case, it can be seen from Eq. (12) that the total increased cost of buyer $i$ in the co-ordinated system becomes

$$\frac{1}{2} \left[ \left( \frac{1 - \frac{T_i}{k_i T}}{k_i} \right)^2 h_i^2 + \frac{h_i^2}{k_i} \right] k_i T = \frac{1}{2} \left[ \left( \frac{1 - \frac{T_i}{k_i T}}{k_i} \right)^2 k_i T \right] d_i h_i^2 + \frac{1}{2} \left[ \left( \frac{1 - \frac{T_i}{k_i T}}{k_i} \right)^2 h_i^2 \right] d_i k_i T.$$  

(16)

In order to entice the buyers to participate in the co-ordination, the vendor needs to devise some mechanisms to compensate the increased cost of (16). The first term of (16) is the increased capital opportunity cost, while the second term is the increased storage cost. The capital opportunity cost can be regarded as a time value of money, the vendor can offer the buyers a delayed payment period, $M$, to compensate this increased capital opportunity cost, where

$$M_i = \frac{1}{2} \left( 1 - \frac{T_i}{k_i T} \right)^2 k_i T. \quad \text{(Note that $k_i T$ is the ordering cycle)}$$  

(17)

Hence, the vendor is required to bear an extra holding cost of

$$\sum_{i=1}^{n} \sum_{i=1}^{n} h_i^2 d_i \left( \frac{1 - \frac{T_i}{k_i T}}{k_i T} \right)^2 k_i T = \frac{1}{2} \sum_{i=1}^{n} h_i^2 d_i \left( k_i T + \frac{T_i^2}{k_i T} - 2T_i \right).$$  

(18)

Note that in Eq. (18), the only information required is the buyer’s order interval under the independent policy ($T_i$) that can be obtained through ordering histories.

In addition, the increased storage cost of buyer $i$ that has to be paid by the vendor is

$$h_i = \frac{1}{2} \left( 1 - \frac{T_i}{k_i T} \right)^2 h_i^2 \quad \text{d} k_i T.$$  

(19)

Note that the only cost information required in Eq. (19) is $h_i^2$, which the vendor can obtain from the market.

With the above two incentive mechanisms, namely the delayed payment period and the storage cost sharing, each of the buyers will not be worse off when compared to the individual cost under the independent policy. The vendor’s extra cost is thus expressed as a summation of (18) and (19)

$$\frac{1}{2} \sum_{i=1}^{n} \left( h_i^2 + h_i^2 \right) d_i \left( k_i T + \frac{T_i^2}{k_i T} - 2T_i \right).$$  

(20)

Therefore, the vendor’s total relevant cost in the co-ordinated system becomes

$$V^s = \left[ \frac{S_v}{NT} + \frac{h D}{2} \left( \frac{h D^2}{2P} \right) \right] N T + \sum_{i=1}^{n} \left[ \frac{C_i + 0.5 h_i^2 h_i^2 d_i T_i^2}{k_i T} + d_i \left( \frac{h D}{2} - \frac{1}{2} (h - h_i^2) \right) \right] k_i T - \left( h_i^2 + h_i^2 \right) d_i T_i^2.$$  

(21)

As the total relevant cost of each buyer in the co-ordinated system is kept as the same as that of independent optimization, our objective is to determine the “optimal” values of $N$ and $k_i$’s that minimize (21).

In this paper, we arbitrarily set $T = 1$. Since the $k_i$’s are functions of $N$ or vice versa, we may not be able to optimize the two decision variables simultaneously. So we use the cost function (21) to find the $k_i$ for a given $N$.

For fixed $N$ and $T = 1$, Eq. (21) is solely a function of $k_i$ and can be expressed as

$$V^s(k_i) = \sum_{i=1}^{n} \left( \frac{W_{1i}}{k_i} + W_{2i} k_i \right) + W_{3i}$$  

(22)

where

$$W_{1i} = C_i + 0.5 h_i^2 + h_i^2 d_i T_i^2.$$  

$$W_{2i} = d_i \left( \frac{h D}{2} - \frac{1}{2} (h - h_i^2) \right)$$  

and

$$W_{3i} = \frac{S_v}{N} + \frac{h D}{2} \left( \frac{h D^2}{2P} \right) N - \left( h_i^2 + h_i^2 \right) d_i T_i^2.$$  

By taking the first derivative, $(dV^s/dk_i) = -(W_{1i}/k_i^2) + W_{2i}$, it can be shown that when

$$W_{2i} = d_i \left( \frac{h D}{2} - \frac{1}{2} (h - h_i^2) \right) > 0,$$

the vendor’s total relevant cost above is a convex function of $k_i$ and we need to search for the minimum point. However, if

$$W_{2i} = d_i \left( \frac{h D}{2} - \frac{1}{2} (h - h_i^2) \right) \leq 0,$$

the vendor’s total relevant cost is a monotone decreasing function of $k_i$ and the optimal solution should be as large as possible, i.e. $k_i = N$.

Here, we develop an algorithm to determine the “near-optimal” solution:

**STEP 1:** Set $N = 1$ and $T = 1$

**STEP 2.1:** Determine all factors of $N$.

**STEP 2.2:** IF $d_i \left( \frac{h D}{2} - \frac{1}{2} (h - h_i^2) \right) > 0$, GO TO STEP 2.3 ELSE set $k_i = N$ and GO TO STEP 3

**STEP 2.3:** Calculate $\Phi = \frac{C_i + 0.5 h_i^2 + h_i^2 d_i T_i^2}{2P} - \frac{\left( h - h_i^2 \right)}{2P}$. 

**STEP 2.4:** Find $k_i$ such that $\Phi \leq k_i$ or $\Phi > k_i$ (See Appendix I for the derivation of the inequalities).

**STEP 2.5:** IF $k_i \geq N$, set $k_i = N$ and GO TO STEP 3 ELSE IF $k_i$ is a factor of $N$, then GO TO STEP 3 ELSE Find two consecutive factors of $N$, $k_i^1$ and $k_i^2$, such that $k_i^1 < k_i^2$.

**STEP 2.6:** Substitute $k_i^1$ and $k_i^2$ into the function.

$$f(k_i) = \frac{C_i + 0.5 h_i^2 + h_i^2 d_i T_i^2}{k_i T} + d_i \left( \frac{h D}{2} - \frac{1}{2} (h - h_i^2) \right) k_i T$$

and set $k_i = k_i^1$ if $f(k_i^1) < f(k_i^2)$, otherwise set $k_i = k_i^2$.

**STEP 2.7:** Substitute $N$ and the $k_i$’s into (21) to obtain the vendor’s cost.

**STEP 3:** IF $N < 365$, set $N = N + 1$ and repeat STEP 2.1–2.7 ELSE GO TO STEP 4.

**STEP 4:** Take the $(n + 1)$-tuple, $(N, k_1, k_2, ..., k_n)$, which gives the least vendor’s cost as the “optimal”.

5. An equitable scheme on profit sharing

To implement the co-ordination policy proposed in Section 4, the vendor acts as the leader in the supply chain by offering an incentive consisting of a delayed payment period and a storage cost payment to motivate each buyer to change its ordering interval from $T_i$ (independent optimization) to $k_i T$ (co-ordination model). Under the proposed policy, the vendor does not require the buyers to reveal their cost information in determining the values of $k_i$, the lengths of the delayed payment periods and the amounts of the storage cost payments. Hence, on the one hand the proposed policy is easy to implement, and on the other hand it overcomes the major limitation of models in the literature as commented by Chang et al. (2008). With the incentive offered,
buyers’ may be willing to accept the new ordering interval as their costs would not be increased by the co-ordination. However, in reality, the buyers may realize that the vendor gets all the net savings achieved by the co-ordination and hence will strive for the distribution of the savings, i.e. the buyers would force the vendor to share the saving with the buyers. In this case, the vendor may consider an additional incentive scheme, e.g. price discount, to enhance the co-ordination. With the price discount, the purchasing costs of the buyers are reduced and hence a win-win situation can be achieved by the co-ordination. However, this may not result to an equitable sharing of the saving as it is likely that the vendor retains most of the saving when designing the price discount scheme. In this regard, we propose the following equitable scheme on profit sharing that guarantees equal saving percentage to all the parties as compared to the cost under the independent policy:

\[
\text{Let } B_{i}^{\text{indep}} \text{ be the cost of buyer } i \text{ under the independent policy, } V_{i}^{\text{indep}} \text{ and } V_{i}^{\text{coop}} \text{ be the cost of the vendor under the independent policy and the co-ordination model, respectively. The proportional reduction in the total cost of the supply chain as a result of the co-ordination is given by}
\]

\[
\frac{V_{i}^{\text{indep}} - V_{i}^{\text{coop}}}{V_{i}^{\text{indep}} + \sum B_{i}^{\text{indep}}}
\]

To maintain fairness, the proportion of savings allocated (compared to independent policy) to each party should be the same and thus, after reimbursement by the vendor, the cost of the \(i\)th buyer is

\[
\text{Table 1}
\]

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<th>(A_i)</th>
<th>(h_i)</th>
<th>(T_i)</th>
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\text{Table 2}
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</table>

\[
\text{C.K. Chan et al. / Int. J. Production Economics 127 (2010) 95–102}
\]
and the cost of the vendor is

\[ V_{\text{IND}} = \left( V_{\text{IND}} - V_{\text{VS}} \right) + \left( V_{\text{IND}} + \sum V_{\text{BIND}} \right) \]

It can be seen from (24) and (25) that the proposed profit sharing scheme requires the vendor and buyers to reveal their cost information to each other so as to work out the fair proportion of saving and the amounts of reimbursements.

6. Results

In this paper, we present three sets of data for numerical experiments. Example 1 (5-buyer) is excerpted from Banerjee and Burton (1994). The data of Examples 2 (30-buyer) and 3 (50-buyer) are randomly generated. The 2nd to 5th columns of Tables 1–3 depict the demand rates and the cost parameters of the three examples, respectively. As illustrated in Chan and Kingsman (2007), the synchronized cycles model performs better than the independent policy over the whole range of \( D/P \) (0.1 to 0.9). Hence, for simplicity, we arbitrarily fix \( D/P=0.5 \) in all of the three examples. Assume that the storage cost is proportional to the holding cost with factor \( a \), the buyer's and the vendor's storage and capital costs can be expressed as

\[ h^i = z h_i, \quad h^v = (1-z) h_i, \quad h^c = z h_c, \quad \text{and} \quad h^v = (1-z) h_v. \]
In this regard, Eq. (21) is reformulated as

\[
V^s = \left[ \frac{\tau}{NT} \right] \left( \frac{b D}{2} \right) \left( \frac{h D}{2P} \right) \left( \frac{NT}{hD} \right) + \sum_{i=1}^{NT} \left( C_i + 0.5h(1-y) + xz \right) d_i T_i \left( \frac{b D}{2} \right) \left( \frac{h D}{2P} \right) \left( \frac{NT}{hD} \right) \left( 1 - \frac{T_i}{T} \right) \left( (1-y) + xz \right) \left( \frac{d_i T_i}{hD} \right)
\]

As pointed out in Axsäter (2006) that the capital cost is considered to be a dominating part of the holding cost, therefore we set \( \tau = 0.3 \) as an illustration for the numerical examples. That is, 30% of the holding cost is the actual storage cost and 70% is the capital opportunity cost. The three sets of data cover a wide range of demands and cost parameters.

The ordering interval of each buyer and the associated buyer’s cost under the independent policy are tabulated in columns 6 and 7, respectively, of Tables 1–3. For the proposed synchronized co-ordination model with delayed payment, the results are listed in columns 8 and 9. Since each buyer’s cost is maintained the same as under the independent policy, the values in column 7 are the same as those in column 9. The value of \( r_i \) can be obtained via the expression \( r_i = \left( \frac{1 - \tau}{\tau} \right)^2 \) and is shown in column 10. The values of \( r_i \) reflect the degree of cost increment under the synchronized cycles co-ordination as compared to the cost of under the independent policy. For the 5-buyer case, the minimum and maximum values of \( r_i \) are 0.217 and 0.444, respectively; for the 30-buyer and 50-buyer cases, the minimum and maximum values are 0.069 and 0.547, and 0.000 and 0.621, respectively. The range of the degree of adjusting the ordering cycles under synchronization increases as the number of buyers increases. Columns 11 and 12 show the values of \( M_i \) and \( H_i \) which can be calculated by (17) and (19), respectively. From the tables, the length of the delayed payment period, \( M_i \), ranges from 3.25 to 13.33 for the 5-buyer case, 0.35 to 6.53 for the 30-buyer case, and 0.00 to 5.59 for the 50-buyer case. The length of the period offered to the buyers decreases as the number of buyers increases. The observation is promising as it indicates that, for large supply chains, the vendor may not need to offer a relatively long delayed payment periods (as delayed payment period may be considered as “risk” to the buyer) to the buyers in order to entice the buyers to participate in the co-ordination. Recall that the determination of such period does not require any buyer’s cost information.

The production cycle, total buyer’s cost, vendor’s cost, as well as the total system cost are shown in Table 4 for all three examples. The relevant costs under the independent policy serve as the benchmark for evaluating the effectiveness of the co-ordination models. The second row depicts the results of the co-ordination without delayed payment, which is the synchronized cycles model in Chan and Kingsman (2007). Notice that while there is substantial savings for the whole supply chain system, all the savings go to the vendor and the cost borne by the buyers are increased when compared with their independent policy. The buyers, thus, are not willing to participate in the co-ordination. In this paper, we have developed a synchronized co-ordination model with delayed payment that can help to resolve this shortfall. The results are shown in the third row of Table 4 for each example. While offering a delayed payment period and a storage cost sharing mechanism to the buyers, the vendor still gets a significant cost reduction over the independent policy. The vendor obtains a percentage saving of 38.3%, 35.7%, and 32.5%, respectively, for the three examples. For the whole system, the total cost reduction is also significant, i.e. 29.2%, 28.9%, and 23.0%, respectively, for the three examples. As mentioned in Section 4, the buyers are not required to reveal any information regarding the cost parameters, the buyers should be willing to participate in the co-ordination model since they are not worse off. However, the buyers may realize that there are substantial savings to both the vendor and the entire supply chain and hence may request for a share of the total saving obtained. By adopting the profit-sharing scheme in Section 5, all the involved parties are having the same percentage saving as compared to their own incurred cost under the independent policy. The results are shown in the last row with heading “co-ordination with profit sharing” for the three examples. The calculated cost saving for all the parties is now equal to the total system cost saving under the co-ordination model (with delayed payment) without profit-sharing. There is no coincidence that the “calculated” saving is the same as the total system saving. This overall “equitable” saving should eventually equal to the total saving obtained that the saving percentage is less than the vendor’s saving percentage under the co-ordination model without profit-sharing. Further, based upon our numerous simulations (with a wide range of randomly generated cost parameters), the total saving percentage of the whole system is always less than the vendor’s saving percentage and thus an equitable profit sharing scheme always exists. However, a short-comings of this profit-sharing scheme is that the buyers and the vendor are required to reveal the cost information to each other.

7. Conclusions

This research proposes a co-ordination mechanism that incorporates a delayed payment method with a cost-sharing scheme which can guarantee that a buyer’s total relevant cost of co-ordination will not be increased when compared with independent optimization. The delayed payment method compensates the buyer’s increased capital opportunity cost while the increased storage cost of the buyer is shifted to the vendor via the cost-sharing scheme. The major

<table>
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<tr>
<th>Example</th>
<th>Policy</th>
<th>Production cycle</th>
<th>Buyer’s cost</th>
<th>Vendor’s cost</th>
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characteristic of this co-ordination model is that the vendor does not need any of the buyer’s cost information to motivate the buyers to change their policies to allow the savings from co-ordination to be achieved. That is, in the optimization procedure, the only buyer cost information required is the storage cost of each buyer, \( h_i \). However, this value can be obtained from the market. The proposed model also further improves the total system cost when compared with the co-ordination model without delayed payment. In addition to the delayed payment period and storage cost payment, the vendor can enhance the co-ordination by offering price discounts to the buyers so as to achieve a win-win situation. Finally, the profit-sharing scheme proposed by this paper allows an equitable sharing of the system saving.

As in the case of most of the co-ordination models in the literature, the major limitation of our proposed model is that we only consider one product and its demand is deterministic. This may fail to capture the essence of real supply chain situation. For further research, it is promising if we can develop a model which considers simultaneously a number of products with demands varying with time or prices of the products. Another direction for the extension would be developing an equitable profit-sharing scheme which does not require the vendor to acquire any of the buyers’ cost information.

**Acknowledgements**

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**Appendix I**

Consider Eq. (22) and suppose \( k_i = k_i^* \) is the optimal ordering interval of buyer \( i \). Since \( k_i = k_i^* \) yields a minimum value of \( V_i \), it follows:

\[
\Delta V_i(k_i) = V_i(k_i^*) - V_i(k_i - 1) = -\frac{W_{i1}}{k_i^* (k_i^* - 1)} + W_{2i} \leq 0
\]

which yields \( k_i^*(k_i^* - 1) \geq \frac{W_{i1}}{W_{2i}} \), and similarly

\[
\Delta V_i(k_i + 1) = V_i(k_i + 1) - V_i(k_i) = -\frac{W_{i1}}{k_i^* (k_i^* + 1)} + W_{2i} \geq 0
\]

which yields \( k_i^*(k_i^* + 1) \geq (W_{i1}/W_{2i}) \).

Let

\[
W_{i1} \quad \frac{C_i + 0.5(h^2 + h^2) d_i T_i^2}{d_i (\frac{h}{2} D - \frac{1}{2} (h - h^2 + h^2))} = \Phi_i,
\]

and we obtain the inequalities in Step 2.4 as

\[ k_i^*(k_i^* - 1) \leq \Phi \leq k_i^*(k_i^* + 1). \]