Reducing the mean supply delay of spare parts using lateral transshipments policies
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ABSTRACT
Lateral transshipment has been studied lately as a promising policy for increasing the performances of multi-echelon spare parts inventory system. By lateral transshipment spare parts can be moved from one location with excess inventory to another location, at the same echelon, in shortage, with the aim of reducing supply delays of spare parts. This paper will examine the relative effectiveness of two lateral shipments approaches in reducing the mean supply delay (MSD) of a non-repairable item, with respect to a classical policy of no lateral shipments. A simulation model of a two echelon supply network has been implemented and an experiment has been performed by varying different parameters of the supply network, such as the number of warehouses (locations at the lower echelon), the supply lead time from the central depot, the spare parts demand uncertainty, and the size variability of the warehouses. Results show appreciable reductions of MSD when lateral shipments are allowed with respect to the classical policy, in almost every network configuration.

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1. Introduction
The improvements in the information technology coupled with the substantial reduction in the cost of processing, storing, and analyzing data have made sharing of inventories more attractive. Furthermore, logistics companies (and in particular express carriers) have made the rapid movement of parts from one place to another possible and more affordable.

For this reason, the possibility to share stocks between those locations, in the supply chain, that have to face the final demand (through transshipments from locations still having adequate stock levels towards locations that are in shortage), is getting more and more attractive in order to reach a high service level and at the same time to limit total costs.

There are numerous examples of sharing inventories among locations in industrial, retail, and military settings. For example, automotive dealers share parts on a daily basis so that they can complete the repairs of their customer's vehicles quickly; and lateral re-supply of parts among military bases within a geographic region occurs on a regular basis as well.

Numerous researchers have developed analytical models to examine inventory pooling of spare parts. The great part of this literature found in studies on repairable or recoverable items. The main stream of these studies starts by the well known METRIC model of Sherbrooke (1968). In this model, one-for-one stock replenishments (i.e. unit-sized customer demands and replenishments) are considered; this assumption is justified when we deal with repairable slow-moving and expensive items, as in Lee (1987), Axsäter (1990), Grahovac and Chakravarty (2001), Wong et al. (2005).

As far as non-repairable items concern, they are usually considered as consumable items, and transshipment policies are inserted in classical continuous and periodic review replenishment policies. Needham and Evers (1998), Evers (2001), and recently, Xu et al. (2003), Axsäter (2003), Minner and Silver (2005) and Olsson (2009) consider continuous review, order point-order quantity policies. Other studies assume periodic review policies. Examples are Cohen et al. (1986), Karmarkar (1987), Tagaras and Cohen (1992), Tagaras (1989, 1999), Robinson (1990), Archibald et al. (1997), Rudi et al. (2001), and Herer et al. (2002). Solving transshipment problems to optimality is difficult, unless several simplifying hypothesis are assumed (such as, as above mentioned, unit-sized customer demands and replenishments, identical or limited number of retailers, negligible replenishment lead time, etc.). For this reason, some heuristics have been recently proposed in order to provide rules, that incorporate relevant factors of the problem, to find conditions under which it make sense to transship a certain number of units shortages from one retailer to another (Evers, 2001; Minner et al., 2003; Lee et al., 2007; Archibald, 2007; Archibald et al., 2009).

Most of the studies on consumable items concern emergency transshipment, which means that shipments between locations...
can occur only when a shortage happens. In this case, shipments are usually assumed to be fast enough to satisfy the location in shortage. From another view, transshipment could be used to balance the inventory level of different locations at the same echelon before that a shortage happens (Diks and De Kok, 1996; Jönsson and Silver, 1987; Bertrand and Bookbinder, 1998; Banerjee et al., 2003). On the analogy of emergency transshipment, we call this latter situation ‘preventive’ transshipment.

This paper presents a model of a two echelon inventory system, where replenishments from the central depot occur on the basis of a periodic review policy. Two different lateral preventive transshipment policies and one no lateral shipment policy are compared through a simulation study under different network configurations. Several simulations have been performed by varying different parameters of the supply network, such as the number of warehouses (locations at the lower echelon), the supply lead time from the central depot; the spare parts demand uncertainty, and the order size variability of the warehouses, for a total of 72 different network configurations.

The item is a non-repairable spare part, and, with respect to other studies on inventory pooling of consumable goods, some considerations on performance parameters have to be done.

Studies on consumable goods try to minimize a total cost function, which typically includes: carrying inventory costs, replenishment costs (from the central depot), transshipment costs between locations and stock-out costs. In particular, as far as stock-out concerns, demand out of stock of a consumable good is usually considered lost or backordered; and, as a consequence, a fixed cost per stock-out occasion or a specified fraction of the unit value per unit short are considered in the cost function. This implies that the minimization of stock out occasions or the minimization of units of shortage is among the main objectives.

On the contrary, when we deal with a spare part, we have to explicitly consider that each unit short results in a machine (or, more in general, a system) being idled; thus, we have to concentrate our attention on the duration of the shortage, which directly affects machine availability. Any consideration on costs, in this case, has to start from the effect of the transshipment policy on this availability. For this reason, we will evaluate the three different policies considered in our model on the basis of the mean supply delay (MSD) of the item, which directly affects operational availability.

In the next section, the relation between MSD and operational availability is shown. In Section 3, the two-echelon inventory system is described, and the lateral-shipment policies are explained. The model of network configuration and lateral re-supply adopted is based on Banerjee et al. (2003), but the analysis is extended to investigate the effect of lateral shipments on the MSD of a spare parts. Authors find that this model, that considers most significant variables of the problem, is very suitable for the impact evaluation of different transshipment policies on the system performances. Due to its complexity, the problem is hard to be approached analytically. For this reason, we have performed our analysis through simulation experiments, as also Banerjee et al. (2003) did. However, the simulation study presented in Banerjee et al. (2003) is lacking in the validation phase. In effect, the model validation (described in Section 4) allowed us to find out some inaccuracies in the simulation experiment results presented in Banerjee et al. (2003), and to corroborate the credibility of the simulation study results presented herein. These results are discussed in Section 5.

2. Supply delay and availability

In this section, we want to outline how lateral shipment policies, as in general any replenishment policy, affect the availability of a system through its influence on the MSD of spare parts.

The word ‘availability’ can be combined with three different adjectives: inherent, achieved, and operational (Sherbrooke, 2004). Inherent availability is a measure of hardware reliability and maintainability and do not concerns spares.

Inherent availability = \( \frac{100 \cdot MTBF}{MTBF + MTTR} \) \hspace{1cm} (1)

where \( MTBF \) is the mean time between failures and \( MTTR \) is the mean time to repair.

Achieved availability is an improvement over inherent availability, but it is a similar measure that relates to hardware considerations and excludes spare delays.

Achieved availability = \( \frac{100 \cdot MTBM}{MTBM + MCTM + MPMT} \) \hspace{1cm} (2)

where \( MTBM \) is the mean time between maintenance, \( MCTM \) is the mean corrective maintenance time, and \( MPMT \) is the mean preventive maintenance time. The \( MTBM \) may be smaller than the \( MTBF \), because it makes allowance for periods when the system will not be available due to preventive maintenance activities.

The third type of availability is operational availability, and includes the concept that a system is operational if it is not down for either maintenance or supply.

Operational availability = \( \frac{100 \cdot MDT}{MTBM + MCTM + MPMT} \) \hspace{1cm} (3)

where \( MDT \) is the mean downtime due to spares, maintenance (corrective and preventive) and other delays resulting from maintenance action.

Operational availability can be divided into two availability components, for calculation purposes

Maintenance availability = \( \frac{100 \cdot MTM}{MTBM + MCTM + MPMT} \) \hspace{1cm} (4)

that is equivalent to achieved availability, and

Supply availability = \( \frac{100 \cdot MSD}{MTBM + MSD} \) \hspace{1cm} (5)

where \( MSD \) is the mean supply delay.

The \( MDT \) in Eq. (3) is the sum of the delay times in Eqs. (4) and (5)

\[ MDT = MCTM + MPMT + MSD \] \hspace{1cm} (6)

If either maintenance availability or supply availability is high, then their product is an acceptable approximation of operational availability. Maintenance availability is independent of the stockage policy and can be calculated once the maintenance manning, test equipment, and preventive maintenance policy have been defined. Supply availability is independent of the maintenance policy, is a function of the replenishment policy, and is directly affected by the MSD.

By reducing the MSD, that is the only parameter influenced by replenishment and transshipment policies, the mean downtime due to spares decreases, the supply availability increases and, finally, operational availability increases. This gives us the reason for selecting the MSD as the main performance indicator of our simulation study, which will be described later in Sections 3 and 4.

3. Model description

In the following, a 2-echelon inventory system of a non-repairable spare part is considered (see Fig. 1).
At the lower echelon there is a certain number of warehouses that face spare parts demand. These warehouses form a pool that can share inventories via lateral transshipments.

A periodic review replenishment policy \((R, S)\), or order up to level, is adopted for replenishments from the supply depot. The review period \(R\) is much greater than the transportation lead time \(L\) from the depot to each warehouse; this is certainly the case for most service parts. We assume the same \(R\) and \(L\) for all warehouses with \(R \gg L\), and \(L\) not trivial in length; for instance, \(L\) may be several days or a week, where \(R\) may be many weeks or months. In the following, \(R\) will be considered equal to 20 days and \(L\), considered deterministic, can be equal or to 0 or to 2 days.

No safety stocks are considered, in order to emphasize the relative effectiveness of lateral shipments policies.

Spare parts are also managed independently so that we may analyze them one at a time. For the sake of simplicity, only one type of spare parts is considered in the following.

The item daily at the \(i\)-th warehouse during each day is denoted by \(D_i\). It is supposed to be stochastic and stationary, uniformly distributed with mean value equal to \(\overline{D_i}\) (see the next section for lower and upper bounds). Demands are assumed to be independent across warehouses. Backorders are allowed: if a part of the daily demand is not met, it will be satisfied as soon as new items arrive to a warehouse.

Decision parameters considered are:

- The number of warehouses, \(NOW\);
- The average order size variability of the warehouses, \(OSV\), related to different demand rates;
- The uncertainty in the spare parts demand \(Du\);
- The transportation lead time \(L\) from the supply depot to the warehouses;
- The lateral shipment policy considered.

In the simulation experiment, described in Section 4, three levels of \(NOW\) (2, 4, and 8), 2 levels of \(OSV\), 2 levels of \(Du\), 2 levels of \(L\) (0 and 2) and 3 lateral shipment policies are considered.

### 3.1. Degree of demand uncertainty (\(Du\))

The daily demand probability distribution function for a warehouse \(i\) is reported in Fig. 2.

Demand is uniformly distributed around the average value \(\overline{D_i}\). In the low level of uncertainty considered the interval of the uniform distribution is \([1−0.333\overline{D_i}(1+0.333\overline{D_i})]\) while in the high level the interval is \([1−0.667\overline{D_i}(1+0.667\overline{D_i})]\). In Fig. 2, the parameter \(Du\) has been introduced: if demand uncertainty is high, \(Du=2\); if demand uncertainty is low, \(Du=1\).

### Table 1. Mean values of daily demand when OSV is high.

<table>
<thead>
<tr>
<th>Mean daily demand</th>
<th>Number of warehouses (NOW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\overline{D_i})</td>
<td>20, 20, 20</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>40, 26.67, 22.86</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>33.33, 25.71</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>40, 28.57</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>31.43</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>34.29</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>37.14</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>40</td>
</tr>
<tr>
<td>(\overline{D_i})</td>
<td>30, 30, 30</td>
</tr>
</tbody>
</table>

### 3.2. Average warehouse order size variability (\(OSV\))

Two levels of \(OSV\) are adopted, one termed ‘high’ and the other one ‘low’. The average order size for the \(i\)-th warehouse is \(\overline{D_i}R\) units, since it is the product of the average demand \(\overline{D_i}\) per period (day) and the review period \(R\). The order up to level is equal to \(S_i = \overline{D_i}(R+L)\).

In the case of low \(OSV\) each warehouse expected order quantity is equal to 600, implying an identical average demand of 30 units and an order up to level equal to 660 units in the case of \(L=2\), and equal to 600 in the case of \(L=0\).

In case of high \(OSV\) the largest expected order quantity is the double of the smallest, with the remaining order size equally spaced between 400 and 800, following the formula:

\[
\overline{D_i}R = 400 + (i-1)\frac{400}{NOW-1} \quad i = 1, \ldots, NOW
\]

with \(\overline{D_i}R = 400, \overline{D_{now}R} = 800\). \(\overline{D_i}\) can be obviously computed as \(\overline{D_i}R/R\). Table 1 shows mean daily demand for different number of warehouses when \(OSV\) is high.

Note that the average value of mean daily demands \(\overline{D} = (\sum_{i=1}^{NOW}\overline{D_i})/NOW\) is equal to 30 for each combination of \(NOW\) and \(OSV\) (see Eq. (13)). This result will be used later to validate the model (Section 4).

### 3.3. Lateral shipment policies (LSP)

Two policies of lateral shipments and one policy of no lateral shipments are considered.

Lateral shipments policies are based on this consideration: when a warehouse inventory level, at the end of the day, falls below \(\overline{D_i}\), there is a consistent probability of a stock-out in the following day (more than 50%). This probability may be reduced by a transshipment, supposing that the shipment from another
warehouse can be performed within the beginning of the next day (i.e. transshipment lead-times are assumed to be zero).

The triggering condition for the two transshipment policies is reached when, at a particular time \( t \), stock level \( I_i(t) \) of at least one warehouse, e.g., the \( i \)th warehouse, satisfies the condition

\[
I_i(t) < \overline{D}_i(t)
\]  

(8)

Lateral shipments are usually faster than shipments from the central depot, which require picking and handling procedures that, for few items, can be hard to perform and time consuming. Another advantage of using lateral shipments instead, for example, of shipments from the central depot during the review cycle, is that shipping every day from the central depot does not balance inventories among warehouses, but rather implies the introduction of a higher number of items in the system, with consequences on carrying costs. On the contrary, lateral shipments provide a way to compensate demand variability without introducing new (and unnecessary) items in the system.

The three shipment policies adopted are outlined below.

### 3.3.1. No lateral shipments (NLS)

Spare parts can be supplied only by the central depot, on the basis of the periodic review \((R, S)\) policy above described. At the end of the review period \( R \), the quantity to be shipped is determined so that the inventory position of each warehouse reaches the order up to level \( S \). Orders arrive after \( L \) periods. Total backordering is allowed. Orders during the review period are not allowed.

### 3.3.2. Lateral transshipments based on availability (TBA)

As far as supplies from central depot concern, the supply policy is the same as the NLS policy.

Suppose that the triggering condition is reached at a time \( t \), and that there are a number \( J \) of warehouses that satisfies the transshipment order condition \( I_j(t) < \overline{D}_j(t) \) \((j = 1, \ldots, J)\). The expected shortages at the end of the following day for each warehouse \( j \), estimated at time \( t \), are \( S_{Hj}(t) = \overline{D}_j - I_j(t) \), for \( j = 1, \ldots, J \). The aim of transshipments, in this policy, is to re-supply exactly the \( S_{Hj}(t) \) quantity in order to reach the \( \overline{D}_j \) level. Thus, \( S_{Hj}(t) \) also indicates the respective needed transshipment quantities. Note that the TBA policy, as far as lateral shipments concern, is similar to an \((S - 1, S)\) policy, where the order up to level \( S \) is equal to the daily demand rate.

If, at the same time \( t \), there are other warehouses that have more than their expected inventory, they can supply those in shortage. Since \( R \), the review period, is deterministic, the expected inventories on hand at time \( t_c \) for a generic \( i \)-th warehouse is

\[
E[I_i(t_c)] = \overline{D}_i(t_c - t_c)
\]

(9)

where \( t_c \) is the next known scheduled time of receipt of the next shipment from the central depot (see Fig. 3). Note that Eq. (9) represents the expected inventories on hand in case of no lateral shipments. However, in our case, lateral shipments do not substantially affect the average inventory level of a warehouse, because each warehouse has the same probabilities of receiving and shipping items.

The warehouses from which transshipments may be made to the shortage locations are those \( K \) locations with \( I_k(t) > E[I_k(t)] \) \((k = 1, \ldots, K)\), and the amount available for lateral shipments at each of these locations is equal to the quantity in excess, that is \( A_k(t) = I_k(t) - E[I_k(t)] \), \((k = 1, \ldots, K)\).

When the transshipment condition is triggered, transshipment quantities are allocated following this procedure:

i. shortage quantities \( S_{Hj}(t) \) for \( j = 1, \ldots, J \) and excess quantities \( A_k(t) \) for \( k = 1, \ldots, K \) are computed.

ii. The \( K \) warehouses are ranked following the magnitude of the excess stock and are labelled as \( 1', 2', \ldots, K' \) with \( A_1' \geq A_2' \ldots \geq A_K' \). Analogously, the \( J \) warehouses are ranked

### Table 2

Expected shortages per warehouse.

<table>
<thead>
<tr>
<th>NOW</th>
<th>OSV</th>
<th>L</th>
<th>NLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LoDu</td>
<td>HiDu</td>
</tr>
<tr>
<td>2</td>
<td>High</td>
<td>0</td>
<td>206.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>216.1</td>
<td>432.2</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>206.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>216.1</td>
<td>432.2</td>
</tr>
<tr>
<td>4</td>
<td>High</td>
<td>0</td>
<td>206.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>216.1</td>
<td>432.2</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>206.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>216.1</td>
<td>432.2</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0</td>
<td>206.1</td>
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<tr>
<td></td>
<td>2</td>
<td>216.1</td>
<td>432.2</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>206.1</td>
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<tr>
<td></td>
<td>2</td>
<td>216.1</td>
<td>432.2</td>
</tr>
</tbody>
</table>

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*Fig. 3. Expected inventories on hand for a warehouse*
and labelled following the magnitude of shortages, so that \(SH_1 \geq SH_2 \ldots \geq SH_7\).

iii. The lateral shipment quantity \(Q_{j1}(t)\) at time \(t_i\) that will be shipped from the warehouse with the largest excess stock to the one having the greatest current need is determined, breaking ties if any, using \(Q_{j1} = \min(A_i, SH_1)\).

iv. Once \(Q_{j1}\) has been allocated, the excess and shortage quantities are re-computed (step i) and steps ii and iii are repeated, until condition \(A_i \times SH_1 = 0\) is reached. That is, or all warehouses in shortage have been satisfied, or all the excess stocks have already been allocated.

Lateral shipments activities can occur more than once in a review cycle \(R\), each time the triggering condition is reached. However, no lateral shipments are planned the day before the arrival of the replenishment from the central depot, since there will be the delivery of the ordered quantity in the next day.

Note that Eq. (9) is a corrected version of the formula \(E(l_i(t)) = \overline{D}_t(t_j) + L\) reported in Banerjee et al. (2003). The latter, in effect, is not consistent with the hypothesis of no safety stocks, because the expected inventory level should be equal to zero just before the shipment from the central depot arrives (when \(t_j - t_0 = 0\)).

3.3.3. Lateral transshipment for inventory equalization (TIE)

Under this policy transshipment decisions are based on the inventory equalization among warehouses through a stock redistribution. The signal of this redistribution occurs when, in a generic \(i\)th warehouse, the triggering condition Eq. (8) is reached. At this time inventories are redistributed among the warehouses proportionally to \(\overline{D}_i\), in order to have the ‘equalized inventory level’ \(E(l_i(t))\), reached just after the transshipment activity, is expressed by the formula

\[
E(l_i(t)) = \frac{\overline{D}_i(t_j)}{\sum_{i=1}^{NOW} \overline{D}_i} \left[ \sum_{i=1}^{NOW} l_i(t) \right]
\]  

(10)

Thus, for any location \(j\) with \(E(l_j(t)) - l_j(t) > 0\) the quantity to receive is exactly \(E(l_j(t)) - l_j(t)\). Similarly, for any other warehouse \(k\) with \(l_k(t) - E(l_k(t)) > 0\), the quantity to be shipped out is equal to \(l_k(t) - E(l_k(t))\).

It could be supposed that the activity of stock redistribution can be done by a single vehicle and that each complete tour is counted as a single lateral shipment activity (while in TBA any shipments between any pair of warehouses are considered as a transshipment activity).

The TIE policy assumes that the inventory equalization can be done only one time in each replenishment cycle of \(R\) days, namely, only the first time the triggering condition is reached.

4. Design of experiment and model validation

A complete factorial experimental design involving 5 factors is considered, with 3 level of NOW, 2 levels of OSV, 2 levels of Du, 2 levels of L, and 3 levels of LSP (lateral shipment policies), for a total of \(3 \times 2 \times 2 \times 3 = 72\) experimental conditions. For each condition, 100 simulation runs of 20 cycles of review time \(R\) (equal to 20 days) are considered. For each simulation run statistics are collected after a warm-up period of 5 cycles.

With respect to the simulation experiment described in Banerjee et al. (2003), we increased the number of simulation runs, from 5 to 100, in order to obtain narrower confidence intervals on the average values of the outputs measured.

The overall performances measured are:

1. The average inventory level per warehouse (Table 3).
2. The average number of stock out incidents per warehouse over the 20 cycles (Table 4).
3. The average number of units of shortage per retailer over 20 cycles (Table 5).
4. The average number of transshipment activities per warehouse over 20 cycles (Table 6).
5. The mean supply delay over the simulation run length (Tables 7 and 8).
6. The mean supply delay for shortage items (Tables 9 and 10).

Before analyzing results, some considerations on model validation deserve attention. We validated our model through two widely adopted techniques (see Law and Kelton, 2000). The first one consists in computing exactly, when it is possible and for some combination of the input parameters, some measures of outputs, and using it for comparison. The second one, that is an extension of the first one, is to run the model under simplifying assumptions for which its true characteristics are known and, again, can easily be computed.

In effect, in some of the 72 experimental conditions, certain measures of performance can be computed and used for comparison in order to see if the output is reasonable.

Table 3 shows the average inventory level per warehouse. The case of no lateral shipments (NLS) corresponds to an order point, order-up-to-level (R, S) policy, as described for example in Silver et al. (1998), where \(R\) is the review period and \(S\) is the

<table>
<thead>
<tr>
<th>NOW</th>
<th>OSV</th>
<th>L</th>
<th>NLS</th>
<th>TBA</th>
<th>TIE</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

Table 3 Average inventory level per warehouse (standard deviation between brackets).
order-up-to-level. In this case, the average on hand inventory level for the i-th warehouse \((EOHi)\) is equal to

\[
EOHi = SS + \frac{D_i \cdot R}{2}
\]  

(11)

where SS are safety stocks (equal to 0 in our experiment). If there are different locations, the expected on hand inventory level per warehouse \((EOHPW)\), which is the expected value of the performance measured of Table 3) is then equal to

\[
EOHPW = \frac{1}{NOW} \sum_{i=1}^{NOW} EOHi = R \frac{1}{2} \sum_{i=1}^{NOW} D_i
\]  

(12)

Note that \(D_i\) may assume different values, depending on the number of warehouses and on the OSV (see Table 1). However, the average \(D\) of the mean values \(D_i\) is always equal to 30. Thus

\[
D = \frac{1}{NOW} \sum_{i=1}^{NOW} D_i = 30
\]  

(13)

and

\[
EOHPW = \frac{1}{NOW} \sum_{i=1}^{NOW} EOHi = R \frac{1}{2} D
\]  

(14)

Table 4
Average stock-out incidents per warehouse (standard deviation between brackets).

<table>
<thead>
<tr>
<th>NOW</th>
<th>OSV</th>
<th>L</th>
<th>NLS</th>
<th>TBA</th>
<th>TIE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LoDu</td>
<td>HiDu</td>
<td>LoDu</td>
<td>HiDu</td>
</tr>
<tr>
<td>2</td>
<td>High</td>
<td>0</td>
<td>9.83 (1.87)</td>
<td>8.99 (1.82)</td>
<td>9.86 (2.26)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>9.79 (1.84)</td>
<td>8.89 (1.80)</td>
<td>9.79 (2.31)</td>
</tr>
<tr>
<td>4</td>
<td>High</td>
<td>0</td>
<td>9.85 (1.14)</td>
<td>9.90 (1.15)</td>
<td>9.94 (1.63)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>9.86 (1.12)</td>
<td>9.94 (1.13)</td>
<td>9.92 (1.67)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0</td>
<td>9.88 (0.89)</td>
<td>9.93 (0.85)</td>
<td>10.04 (1.43)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>9.91 (0.87)</td>
<td>9.99 (0.86)</td>
<td>10.10 (1.47)</td>
</tr>
</tbody>
</table>

Table 5
Average units of shortage per warehouse (standard deviation between brackets).

<table>
<thead>
<tr>
<th>NOW</th>
<th>OSV</th>
<th>L</th>
<th>NLS</th>
<th>TBA</th>
<th>TIE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LoDu</td>
<td>HiDu</td>
<td>LoDu</td>
<td>HiDu</td>
</tr>
<tr>
<td>2</td>
<td>High</td>
<td>0</td>
<td>205.22 (59.39)</td>
<td>411.03 (119.19)</td>
<td>165.31 (61.16)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>216.11 (64.05)</td>
<td>433.47 (129.58)</td>
<td>176.99 (66.08)</td>
</tr>
<tr>
<td>4</td>
<td>High</td>
<td>0</td>
<td>205.91 (39.28)</td>
<td>412.59 (78.56)</td>
<td>131.64 (39.41)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>218.02 (41.79)</td>
<td>437.15 (89.23)</td>
<td>140.69 (43.42)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0</td>
<td>206.00 (27.16)</td>
<td>412.63 (54.51)</td>
<td>128.63 (40.29)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>218.38 (28.79)</td>
<td>437.25 (57.21)</td>
<td>115.82 (29.16)</td>
</tr>
</tbody>
</table>

Table 6
Average number of transshipment activities per warehouse (standard deviation between brackets).

<table>
<thead>
<tr>
<th>NOW</th>
<th>OSV</th>
<th>L</th>
<th>NLS</th>
<th>TBA</th>
<th>TIE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LoDu</td>
<td>HiDu</td>
<td>LoDu</td>
<td>HiDu</td>
</tr>
<tr>
<td>2</td>
<td>High</td>
<td>0</td>
<td>0.0 (0.0)</td>
<td>0.0 (0.0)</td>
<td>12.0 (3.2)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>0.0 (0.0)</td>
<td>0.0 (0.0)</td>
<td>13.5 (4.0)</td>
</tr>
<tr>
<td>4</td>
<td>High</td>
<td>0</td>
<td>0.0 (0.0)</td>
<td>0.0 (0.0)</td>
<td>13.0 (3.9)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>0.0 (0.0)</td>
<td>0.0 (0.0)</td>
<td>17.7 (2.9)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0</td>
<td>0.0 (0.0)</td>
<td>0.0 (0.0)</td>
<td>20.7 (3.7)</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>0</td>
<td>0.0 (0.0)</td>
<td>0.0 (0.0)</td>
<td>17.7 (2.7)</td>
</tr>
</tbody>
</table>

For each of the 24 NLS different combinations, the EOHWP, calculated through the Eq. (14), should be equal to 300. Differences between this value and the values carried out by simulations (about 285) are due to the fact that data for statistics of the on-hand inventory level are observed at the end of each day. This causes a lower value of the inventory level that, on the average, is equal to \( \sum_{i} D_i/2 \) for each warehouse, and a consequent lower value of the inventory level per warehouse equal to \( \sum_{i} D_i/2 = 15. \) In the case considered, this implies an average inventory level per warehouse equal to 285.

Table 5 shows the average units of shortage per warehouse. Again, in case of no lateral shipments (NLS), analytical derivation of results can be done. Considering the ith warehouse, if demand during the period \( (R+L) \) is normally distributed, with normal probability density function \( f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} \), the expected shortage per replenishment cycle \( (ESP_{RI}) \) can be computed as \( \text{Silver et al., 1998} \)

\[
ESP_{RI} = \int_{-\infty}^{\infty} (x - \bar{x}) f(x) \, dx = \frac{\text{MAD}}{2} = \frac{\sigma_{R+L+1}}{\sqrt{2\pi}}
\]

In the case considered demand probability distribution for the ith warehouse during \( (R+L) \) days can be approximated, for the ‘central limit’ theorem, to a normal distribution with mean value equal to the sum of daily demand rate \( D_i \) and a variance \( \sigma_{R+L+1}^2 \).
equal to the sum of the variance of the daily demand $\sigma_{Di}^2$.

$$
\sigma_{Di}(R+L) \approx \sqrt{(R+L)\sigma_{Di}^2}
$$

The daily demand variance $\sigma_{Di}^2$ is equal to (see Fig. 2)

$$
\sigma_{Di}^2 = \frac{1}{2(3/3)Du \bar{D}_i} \frac{1}{(x-\bar{D}_i)^2} dx = \frac{Du^2 \bar{D}_i^2}{27}
$$

The cycle (review period plus lead time) demand standard deviation is approximately equal to

$$
\sigma_{Di}(R+L) \approx \sqrt{(R+L)\frac{Du^2 \bar{D}_i^2}{27}} = \frac{Du \bar{D}_i \sqrt{(R+L)}}{27}
$$

The expected units of shortage at location $i$ in one cycle is then equal to

$$
ES_{Di} = \frac{Du \bar{D}_i \sqrt{(R+L)}}{27}
$$

The expected units of shortage at warehouse $i$ in $n$ cycles ($n=20$ in our experiment) is equal to

$$
ES_n = nDu \bar{D}_i \sqrt{(R+L)}
$$

If there are different locations, the expected shortages per warehouse (ESW, which is the expected value of the performance indicator of Table 5) will be equal to

$$
ESW = \frac{1}{NOW} \sum_{i=1}^{NOW} ES_n = nDu \bar{D}_i \sqrt{(R+L)}
$$

and, using Eq. (13)

$$
ESW = nDu \bar{D}_i \sqrt{(R+L)}
$$

As a consequence, we expect neither OSV nor NOW to affect ESPW. As example of calculation, in the case of NOW=2, Du=low (Du=1), OSV=low, L=2

$$
ESW = 20 \cdot 1 \cdot 30 \sqrt{(20+2)} = 216.1
$$

Table 2 shows values of ESPW obtained from Eq. (22), for the NLS policy and each combination of NOW, OSV, L, and Du. As said, neither NOL nor OSV do affect ESPW. Outputs shown in Table 5 are consistent with these results, considering an appropriate confidence interval computable from the standard deviation. On the contrary, outputs reported in Banerjee et al. (2003), although obtained from the same simulation experiment, are not consistent with values reported in Table 2. This could be ascribable or to a trivial misprint or to some error in the simulation model.

In order to check if the transshipments mechanism has been properly modelled, the model has been run under simplifying assumptions, under which some outputs are computable. For example, a simple run with only 2 warehouses, L=0, and deterministic daily demand (equal to 20 for one warehouse and to 40 for the other one) has been performed. In this case, it has been possible to ‘simulate’ one replenishment cycle of 20 days on a spreadsheet, also for TBA and TIE policies, and to compute main outputs. The correspondence of results between outputs collected from the simulation run and those computed through the spreadsheet, confirmed us the validity of our model implementation.

5. Analysis of results

Table 3 shows the average inventory level per warehouse, and indicates that the total stock allocated among all the warehouses at the second echelon is not affected by the choice of a lateral transshipment policy (NLS, TBA, and TIE). This is because, in all the three cases, there wont be any injection of additional stock from the higher echelon.

Tables 4 and 5 show, respectively, the average stock-out incidents and the average units of shortages per warehouse during a 20 cycles simulation run (remember that each of the 100 simulation runs lasts 25 cycles, but statistics are collected after a warm-up period of 5 cycles). From these two tables what comes out is that the implementation of some lateral shipments policy does not affect the probability that a stock-out occurs (stock-out incidents), but rather affects the size of unmet demand quantity each time a stock-out happens. Table 4 shows that the probability, among all warehouses, that a stock-out occurs during a cycle is about 50% (an average stock-out incident per warehouse value around 10 is observed after 20 cycles) for each of the three lateral shipment policies (NLS included). This result was easily predictable for the case of NLS, because, as already stated, this case is equivalent to a periodic review replenishment policy (R, S) without safety stocks.

Table 5 shows that TBA and TIE policies can effectively reduce the number of units of shortage, especially when the number of warehouses increases; furthermore, TIE seems to perform on an average slightly better than TBA. A significant influence of demand uncertainty is, as expected, remarkable, but seems to affect results in the same way for each of the three lateral shipment policies. Supply lead time and order size variability have not an evident influence on the units of shortages.
Table 6 shows transshipment activities per warehouse during a simulation run. Results may be used as reference data to perform a cost analysis (that is not in the aim of this work), remembering that a single transshipment activity in TIE policy implies a complete tour of warehouses. Table 7 shows the MSD. The mean supply delay in each cycle (expressed in days) for a warehouse is equal to the ratio between the sum of negative daily stock levels observed in the cycle and the sum of the daily effective demands \( d_t \), occurred in the same cycle (of 20 days)

\[
MSD = \frac{\sum_{t=1}^{20} d_t}{\sum_{t=1}^{20} l_t}
\]

(24)

Values in Table 7 (average values per warehouse) are very little, because a great part of demand is met during a cycle (the probability to go in stock out become not negligible only in the last few days of each cycle). However, lower values of MSD can be observed when TBA or TIE policies are applied, especially when the number of warehouses increases, with respect to NLS policy.

In order to evaluate the impact, on the mean supply delay, of TBA and TIE policies with respect to the NLS policy, results have been compared under the same network configuration (i.e. same NOW, OSV, L, and Du). For each of these 3 \( \times 2 \times 2 = 24 \) network configuration, a paired \( t \)-test has been performed on the difference in means between MSD(TBA)−MSD(NLS) and MSD(TIE)−MSD(NLS). Naturally, the paired \( t \)-test required that paired observations which were obtained from replications performed with the same random seed (see Law and Kelton, 2000). The null hypothesis is \( H_0: MSD(TBA)−MSD(NLS)=0 \) when TBA and NLS are compared, and is \( MSD(TIE)−MSD(NLS)=0 \) when TIE and NLS are compared. In all the 24 cases, the \( P \)-value of the \( t \)-test resulted lower than 0.0001, indicating that the null hypothesis has always to be rejected and that the alternative hypothesis (respectively, \( H_1: MSD(TBA)−MSD(NLS) \) and \( H_1: MSD(TIE)−MSD(NLS) \) has to be accepted. Table 8 shows the tests results in terms of expected MSD percentage deviation for TBA and TIE policies with respect to the corresponding NLS configuration, with a two sided 99% paired-\( t \) confidence interval. The narrow confidence intervals obtained allow to state some evidences about the reduction of the mean supply delay when lateral shipments are implemented (TBA or TIE), with respect to the case of no lateral shipments. In particular, the MSD reduction

- ranges from about 20% to 65%.
- increases as the number of warehouses increases;
- increases as the demand uncertainty increases;
- is not significantly affected by the order size variability and by the supply lead time;
- is higher for TIE than for TBA policies.

Table 9 shows the MSD only for items that went stock-out during a cycle, i.e., the average time of supply for a spare part that was out of stock. Mean values are just above 1 day per cycle. Similarly to the previous case, a paired \( t \)-test has been performed, in order to evaluate possible reductions provided by the lateral shipments policies. Table 10 shows, again, the percentage deviation for TBA and TIE policies with respect to the corresponding NLS configuration. Also in this case reductions of supply delay can be observed in all the configurations. Evidently, shortages happen later during the cycle of 20 days, when the replenishment from central depot is forthcoming.

However, the reduction is smaller than those observed for the total MSD, and ranges from about 2% to 29%. This means that reductions of total MSD are due to the sum of two beneficial effects produced by the implementation of transshipments: a reduction of the number of units that go out of stock and a reduction of the supply delay when units go out of stock.

6. Summary

This paper has examined the relative effectiveness of two lateral shipments approaches in reducing the mean supply delay (MSD) of a spare part in comparison with a policy of no lateral shipments (NLS). The supply network analyzed has two echelons, and shipments from the central depot are decided in accordance with a periodic review replenishment policy, or order up to level. Different parameters of the supply network have been considered in the simulation study, such as the number of warehouses, the supply lead time from central depot, the spare parts demand uncertainty, and the order size variability of warehouses, for a total of 72 different network configurations.

The two lateral shipment policies analyzed, namely TBA (lateral transshipment based on availability) and TIE (lateral transshipment for inventory equalization) demonstrate their efficacy in reducing the mean supply delay, in comparison with the no lateral shipment policy (NLS), under almost all network configurations. Reductions of MSD range from about 20% to 65%, with respect to the MSD values observed in the analogous NLS configurations. Reduction of MSD is reached by both reducing the number of shortages, and delaying stock-out events during a replenishment cycle.

The conditions in which transshipments provide higher benefits are when uncertainty of the spare part demand is high, and when the number of warehouses of the supply network, between which lateral shipments may take place, is relevant. On the contrary, the order size variability of each warehouse seems not to have significant influence on the reduction of the mean supply delay.

The inventory equalization technique among all warehouses, provided by the TIE policy, seem to perform slightly better than the TBA policy, which is based on ‘peer to peer’ transshipments, between a couple of warehouses at a time.

A possible future investigation, based on this model, could be to relax the assumption of independence between daily demands of warehouses. A negative correlation should very likely accentuate the benefits of transshipments implementation, while a positive correlation should lower it. Further improvements also include the implementation of some variation in the transshipment policies, such as different triggering conditions and different equalization criteria. Performances of transshipments policies could be also compared to those observed in the hypothetic situation of complete pooling of inventories among warehouses.

References


